Real Frequency Technique Applied to the Synthesis of Lumped Broad-Band Matching Networks with Arbitrary Nonuniform Losses for MMIC's

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Abstract —A new computer-aided synthesis technique is presented in this paper for treating the synthesis of lumped matching networks with arbitrary nonuniform losses. It is especially applicable to the design of the broad-band amplifiers in MMIC's. A new, useful theorem and two corollaries are developed for the transformation between lossy or lossless networks and lossless ones, so that the design of the lossy matching networks is considerably simplified relative to [1] and can yield any complex models of the lumped elements with arbitrary nonuniform losses. An example is given to show the general applications of the new method in monolithic broad-band amplifier design.

I. Introduction

With the rapid development of GaAs material and the resolution of many problems regarding device reliability and wafer processing in the last few years, an increasing number of MMIC's have become practical. Matching networks constitute one of the most important parts in the design of MMIC's. They are usually constructed by lumped and/or transmission line elements. In order to obtain a broader bandwidth capability and occupy less GaAs chip area, lumped elements are often preferable. Nevertheless, the losses of lumped elements fabricated on semi-insulating GaAs substrates for MMIC's are so large that the transducer power gain (TPG) changes a great deal. In accounting for these kinds of losses in the design of matching networks, one faces two alternatives. One is to synthesize a lossless matching network first and then substitute lossy element models for the lossless ones. However, this will result in the gain response deviating from the original one at different frequencies. The other is to synthesize a lumped matching network with lossy elements directly. But this would be more difficult for the general case of arbitrary nonuniform dissipation. In 1939, Darlington in his well-known paper [2] used reactive elements which have losses of the semiuniform type (i.e., all inductors have one quality factor Q; all capacitors have another Q) to synthesize a lossy filter. After that, Andersen [3] developed Darlington's work and provided realizability conditions for the semiuniform type, three synthesis procedures, and insertion loss theory. Based on Andersen's theory, Su [4] extended the analytical theory of the broad-band matching network [5]–[7] to the semiuniform case. Though this analytical theory has its theoretical value, it is too complicated to be applied to the practical design of MMIC's. In 1984, Liu [1], using both analytical and CAS techniques, solved the lossy synthesis problem for the case of unequal inductor and capacitor losses (i.e., semiuniform losses) with arbitrary circuit topology and realizable gain functions. Even though this new result was directly applicable to the synthesis of the matching networks for MMIC's, the new theory did not take the parasitic reactances of the lossy circuit elements into account, used only the simplified FET models, and could not synthesize the circuit topology directly!

In order to solve the problem mentioned above, we provide a new method which can directly synthesize the lumped matching networks with arbitrary nonuniform losses for MMIC's. Since the simplified "real frequency" technique (SRFT) [8] is used to synthesize a primary lossless double matching network, our method has all the advantages of the SRFT. With the help of the transformation provided by our theorem, the lossless reactive lumped elements can be replaced by lumped circuit elements with arbitrary nonuniform losses including skin, conductor, and dielectric losses. Therefore, all kinds of losses in the matching network are considered except the possible deviations of the lossy element models from the actual ones. If it is necessary to consider more exact lossy element models to achieve even better performance, a CAD program may be used to modify the element values of the actual lossy L's and C's of the matching network. In conclusion, it can be predicted that the new method we developed for the synthesis of lossy matching networks for MMIC's will have wide applications in the more general cases.

II. Transformation Between Lossy and Lossless Matching Networks

The matching network is the most important part in the design of MMIC's. The classical matching problem is one of constructing a lossless matching network between a

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generator and a load such that the transfer of power is maximized over a prescribed frequency band. But the modern matching problem, especially the matching problem in MMIC's, is one of synthesizing and/or designing a lossy matching network. Thus, the usual approaches to the synthesis of the lossless matching network, such as the analytic gain–bandwidth theory [5]–[7], the commercially available CAD program AMPSYN [9], and the newly developed real frequency technique [10], [8], [11], cannot be used to synthesize the lossy matching network. It is therefore necessary to find a new method which can synthesize the lossy matching network directly.

In his paper [2] Darlington gave some attention to dissipative reactance networks. He presented a general theory (described by Bode) which states that if each element of a network N produces an individual impedance proportional to Z_1 or Z_2, any impedance of the complete network will be the product of Z_2 and a rational function of Z_1/Z_2. That is, if Z_t is a branch impedance of the network N, then it will have one of the following expressions:

$$Z_t = r_t Z_1 \tag{1a}$$

or

$$Z_t = r_t Z_2 \tag{1b}$$

where Z_1 and Z_2 may be any physically realizable impedances of *RLCT* networks which contain resistors, inductors, capacitors, and transformers; r_t is a positive real coefficient. It is easy to verify that any impedance Z of the network N may be written as

$$Z = Z_2 f(Z_1/Z_2) \tag{2}$$

where $f(Z_1/Z_2)$ is a rational function of Z_1/Z_2. To our knowledge, although this important theorem has been used indirectly by several authors [3], [4], [12], [13] to solve the problems of semiuniform lossy networks, general applications of it have not yet been found.

By careful consideration and derivation, it is found that any network, lossy or lossless, can be transformed to a lossless network $\boldsymbol{M}$ if it satisfies the condition of Bode's statement.

Theorem : Suppose each element of a lossy or lossless network N produces an individual impedance proportional to Z_1 or Z_2. Then any impedance of the network, if it exists, can be transformed to a corresponding impedance of the lossless network $\boldsymbol{M}$ by the following transformation (See Appendix I):

$$\tilde{Z} = Z/\sqrt{Z_1 Z_2} = \tilde{Z}_c f(\tilde{Z}_l/\tilde{Z}_c). \tag{3a}$$

In reverse, the transformation is also true. That is,

$$Z = \sqrt{Z_1 Z_2}\,\tilde{Z} = Z_2 f(Z_1/Z_2) \tag{3b}$$

where Z is any impedance of the lossy or lossless network N; $\tilde{Z}$ is a corresponding impedance of the lossless network $\boldsymbol{M}$; $\tilde{Z}_l = \Omega$ is an impedance of the equivalent inductor; $\tilde{Z}_c = 1/\Omega$ is an impedance of the equivalent capacitor; $\Omega = \sqrt{Z_1/Z_2}$ is an equivalent complex angular frequency; and $f(x)$ is a rational function of x.

From the theorem, it is clear that the right-hand side of (3a) has the same form as that of (3b). That is to say, all types of impedance Z_1 or Z_2 can be transformed to the corresponding lossless ones, regardless of whether they possess the properties of semiuniform losses or frequency-dependent losses! Two examples are given below.

Example 1: If all the reactive elements of the network N have semiuniform losses as in [1], we have

$$Z_l = L(s + d_l) = LZ_1 \tag{4a}$$

$$Z_c = 1/[C(s + d_c)] = Z_2/C = 1/(CY_2) \tag{4b}$$

where Z_l and Z_c are any branch impedances corresponding to the semiuniform lossy inductor and capacitor; L and C are the corresponding inductance and capacitance; Z_1 and $Z_2 = 1/Y_2$ are the same as in the theorem; and s is a complex angular frequency. The quantities d_l and d_c are inductor and capacitor losses or dissipation factors and they can be written as

$$d_l = \omega_m/Q_l \tag{5a}$$

$$d_c = \omega_m/Q_c. \tag{5b}$$

Here Q_l and Q_c are the quality factors of the lossy inductor and capacitor, and ω_m is the angular frequency at which the Q's are defined. Then, the corresponding lossless reactive elements have an equivalent complex angular frequency given by

$$\Omega = \sqrt{(s + d_l)(s + d_c)}\,. \tag{6}$$

Example 2: If both skin and dielectric losses are considered, then all the impedances of the lossy circular inductors and MIM (metal–insulator–metal) capacitors [14] may be written as

$$Z_l = L(s + d_l\omega^{1/2}) = LZ_1 \tag{7a}$$

$$Z_c = (1/C)[1/s + \omega^{1/2}/d_c + 1/(d_d\omega)] = Z_2/C = 1/(CY_2) \tag{7b}$$

where d_l and d_c are given by

$$d_l = \omega_m^{1/2}/Q_l \tag{8a}$$

$$d_c = \omega_m^{3/2}/Q_c \tag{8b}$$

and d_d is equal to the quality factor Q_d, which is determined by the dielectric loss of the GaAs material, and the others are the same as those in Example 1. Thus, the corresponding lossless reactive elements have an equivalent complex angular frequency given by

$$\Omega = \sqrt{[s + d_l\omega^{1/2}]/[1/s + \omega^{1/2}/d_c + 1/(d_d\omega)]}\,. \tag{9}$$

If the impedance matrix of the network N exists, then from the theorem we will have the following corollary.

Corollary 1: If any impedance of the network N satisfies the transformation of the theorem, then the impedance matrix of the network N, if it exists, will also satisfy the similar transformation

$$\tilde{\boldsymbol{Z}} = \boldsymbol{Z}/\sqrt{Z_1 Z_2} = \tilde{Z}_c \boldsymbol{F}(\tilde{Z}_l/\tilde{Z}_c) \tag{10a}$$

$$\boldsymbol{Z} = \sqrt{Z_1 Z_2}\,\tilde{\boldsymbol{Z}} = Z_2 \boldsymbol{F}(Z_1/Z_2) \tag{10b}$$

where

$$\tilde{Z} = \begin{pmatrix} \tilde{Z}_{11} & \tilde{Z}_{12} \\ \tilde{Z}_{21} & \tilde{Z}_{22} \end{pmatrix} \qquad Z = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

and

$$F(x/y) = \begin{pmatrix} f_{11}(x/y) & f_{12}(x/y) \\ f_{21}(x/y) & f_{22}(x/y) \end{pmatrix} \quad \begin{matrix} x = \tilde{Z}_l & \text{or} & x = Z_1 \\ y = \tilde{Z}_c & \text{or} & y = Z_2 \end{matrix}$$

Similarly, we can also have transformations between the admittance and its matrix of the network N and the ones of the network M if the conditions are satisfied.

Since the scattering parameters are often used in the design and analysis of microwave circuits, the following corollary is very useful for obtaining the scattering parameters of the lossy matching network N from the ones of the lossless matching network M. It is especially applicable to networks which generally do not possess impedance and/or admittance matrices but can always be represented with the scattering parameters.

Corollary 2: If each element of a lossy or lossless network N produces an individual impedance proportional to Z_1 or Z_2, there will always exist a transformation between the scattering matrix of the lossy or lossless network N and the one of the lossless network M (see the proof in Appendix II):

$$\tilde{S}(\Omega) = \tilde{S}\left(\sqrt{Z_1/Z_2}\right) = \left[(I+S) - \sqrt{Z_1Z_2}(I-S)\right] \cdot \left[(I+S) + \sqrt{Z_1Z_2}(I-S)\right]^{-1} \tag{11a}$$

$$S(s) = S(Z_1, Z_2) = \left[\sqrt{Z_1Z_2}(I+\tilde{S}) - (I-\tilde{S})\right] \cdot \left[\sqrt{Z_1Z_2}(I+\tilde{S}) + (I-\tilde{S})\right]^{-1} \tag{11b}$$

where $S(s)$ is any scattering matrix of the lossy or lossless ntwork N, $\tilde{S}(\Omega)$ is a corresponding scattering matrix of the lossless network M, and I is the identity matrix.

III. Simplified "Real Frequency" Technique Applied to Synthesis of the Lumped Matching Network with Arbitrary Nonuniform Losses

If the transformation between networks N and M is obtained, we can use them in the design of lossy broad-band matching networks. It is known that in the design of lossless broad-band matching networks, the "real frequency" technique introduced by Carlin in 1977 [10] is a very efficient approach, since it is numerical and utilizes only real frequency (e.g., experimental) load and generator data. With the help of this technique, no model or analytic impedance functions for load and generator are necessary, nor is the matching network topology or analytic form of the system transfer function assumed. Since the simplified real frequency technique (SRFT) [8] employs only the lossless scattering parameters to optimize the *TPG* of a lossless matching system with a complex generator and a load, and to realize the lossless matching networks, then it is easier to obtain the scattering parameters of the lossy matching networks from those of the lossless matching networks by the transformation of Corollary 2 and the use of the lossy scattering parameters in the design of the broad-band amplifier with lossy matching networks in MMIC's.

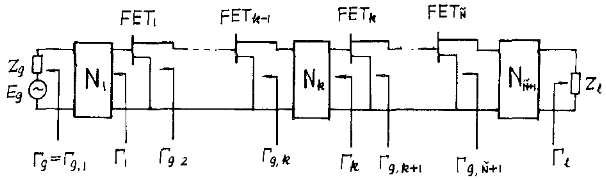

Fig. 1. The multistage FET amplifier used for defining the *TPG*.

Algorithm: Computation of the TPG of the Broad-band Multistage FET Amplifier in MMIC's

First, suppose $\tilde{e}_{11}(s)$, which is a unit normalized input reflection factor of the lossless matching network M, is given by

$$\tilde{e}_{11}(s) = \frac{h(s)}{g(s)} = \frac{h_1 + h_2 s + h_3 s^2 + \cdots + h_{n+1}s^n}{g_1 + g_2 s + g_3 s^2 + \cdots + g_{n+1}s^n} \tag{12}$$

where n specifies the maximum number of reactive elements in M. Then, the unit normalized scattering parameters of M can be obtained by the well-known Belevitch representation [15] and by the assumptions of Yarman and Carlin [8]. However, it should be emphasized that an impedance level transformer, which is often encountered, can usually be avoided by fixing h_1 and h_{n+1} to be zeros for the low-pass and high-pass cases and by using the Norton transformation for the bandpass case. By optimizing the TPG, the coefficients of the numerator polynomial $h(s)$ can be specified by the procedures described in [8].

Second, after the scattering parameters of the lossless matching network M have been obtained, the corresponding ones of the lossy matching network N can be constructed by the transformation of Corollary 2.

Third, defining the transducer power gain for the broad-band multistage FET amplifier is a very important problem. If a better *TPG* is needed, iteration (when the s_{12} of the FET is too large) has to be employed even in a single-stage amplifier, since in the optimization of the input matching network of the single-stage amplifier, the output of the FET is terminated with a unit normalized 1 Ω load instead of the output matching network. According to this design procedure [8], [16], the matching network optimized is not the optimal one. In order to reduce the number of iterations or even eliminate iteration, the output of the FET may be terminated with the complex conjugate output impedance of the FET in the optimization of the *TPG*, for the input impedance of the output matching network optimized usually approaches the complex conjugate output impedance of the FET when the input matching network is connected. The definition of the *TPG* as defined by the authors is given by (referring to Fig. 1 with the details of derivation given in Appendix III)

$$T(\omega) = \frac{\text{power delivered to load } Z_l}{\text{power available from generator } E_g} = \prod_{k=1}^{\tilde{N}+1} T_k \tag{13}$$

where for $k = 2, \cdots, \tilde{N}$,

$$T_k = \frac{\text{power available from FET}_k}{\text{power available from FET}_{k-1}}$$
$$= \frac{\left(1-|\Gamma_{g,k}|^2\right)|e_{21,k}|^2|s_{21,k}|^2}{|1-\Gamma_{g,k}e_{11,k}|^2|1-\Gamma_k s_{11,k}|^2\left(1-|\Gamma_{g,k+1}|^2\right)} \tag{14}$$

in which $\tilde{N}$ is the number of FET's, $\Gamma_{g,k}$ is a real normalized reflection coefficient of the equivalent Thevenin generator at the output of the FET_{k-1}, and Γ_k is a real normalized reflection coefficient at the output of N_k:

$$\Gamma_k = e_{22,k} + \frac{e_{12,k}e_{21,k}\Gamma_{g,k}}{1-e_{11,k}\Gamma_{g,k}}. \tag{15}$$

$\Gamma_{g,k+1}$ is a real normalized reflection coefficient similar to $\Gamma_{g,k}$.

$$\Gamma_{g,k+1} = s_{22,k} + \frac{s_{12,k}s_{21,k}\Gamma_k}{1-s_{11,k}\Gamma_k}. \tag{16}$$

Here $e_{ij,k}$ and $s_{ij,k}$ $(i, j=1,2)$ are the unit normalized scattering parameters of N_k and FET_k. The TPG of the first stage of the amplifier is defined as

$$T_1 = \frac{\text{power available from FET}_1}{\text{power available from generator } E_g}. \tag{17}$$

T_1 has the same expression as for T_k, in which the $\Gamma_{g,1}$ is equal to real normalized reflection coefficient of the generator. The last stage of the amplifier has a TPG defined by

$$T_{\tilde{N}+1} = \frac{\text{power delivered to load } Z_l}{\text{power available from FET}_{\tilde{N}}}$$
$$= \frac{\left(1-|\Gamma_{g,\tilde{N}+1}|^2\right)|e_{21,\tilde{N}+1}|^2\left(1-|\Gamma_l|^2\right)}{|1-\Gamma_{g,\tilde{N}+1}e_{11,\tilde{N}+1}|^2|1-\Gamma_{\tilde{N}+1}\Gamma_l|^2} \tag{18}$$

in which Γ_l is the real normalized reflection coefficient of the load.

From the definition of T_k, it can be seen clearly that T_k is related only to $\Gamma_{g,k}$, $e_{ij,k}$, and $s_{ij,k}$ $(i, j=1,2)$, and has nothing to do with the following networks from N_{k+1} to $N_{\tilde{N}+1}$.

IV. Computer-Aided Design of Broad-Band Amplifier in MMIC's

Based upon the design procedure described above, a new computer program, called SLMNA, has been developed by the authors. In order to optimize the *TPG* of the amplifier, the program can construct the arbitrary nonuniform lossy matching networks of the optimal topologies. The measured scattering parameters of the 0.5 μm low-noise GaAs MESFET in [1] are used to design a broad-band amplifier covering the octave frequency band from 7 to 14 GHz. In the optimization scheme, an ideal flat form of the gain is approximated by employing the least square error functions, and the modified POWELL technique is applied for minimizing the least square error functions. The results obtained are satisfactory. However, it should be pointed out that for many practical cases, it is sufficient to initialize the unknown coefficients $h_i (i=1, \cdots, n+1)$ as $+1$ or -1.

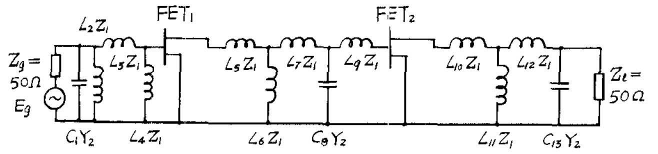

Fig. 2. Design of two-stage amplifier. $C1 = 0.7442$ pF, $L2 = 0.3458$ nH, $L3 = 0.6262$ nH, $L4 = 4.1564$ nH, $L5 = 1.0298$ nH, $L6 = 0.3015$ nH, $L7 = 0.0603$ nH, $C8 = 0.8052$ pF, $L9 = 0.8283$ nH, $L10 = 0.739$ nH, $L11 = 0.4214$ nH, $L12 = 0.0509$ nH, $C13 = 0.5967$ pF, $Z_1 = (j\omega + 11863.53\omega^{1/2})$, $Y_2 = 1/(1/j\omega + \omega^{1/2}/1.30446\times10^{18} + 1/100\omega)$.

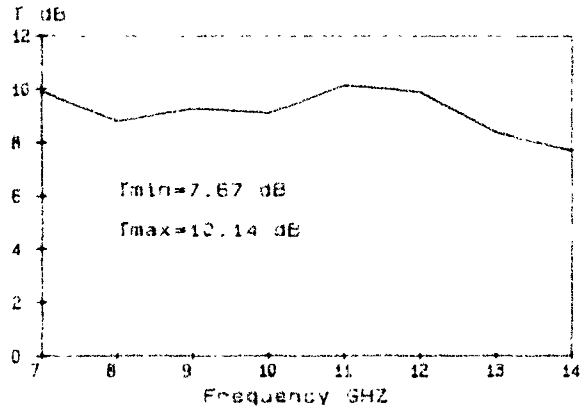

Fig. 3. Performance of the amplifier shown in Fig. 2.

Example

In our example, a two-stage monolithic microwave integrated broad-band amplifier is designed. The lossy element models in (7a) and (7b) are employed in the design of the matching networks of the amplifier. Inputs to the program and the performance of the amplifier are summarized as follows:

- Generator: $Z_g = 50\ \Omega$;
- Load: $Z_l = 50\ \Omega$;
- Skin loss of the inductors: $Q_l = 25$;
- Skin loss of the capacitors: $Q_c = 50$;
- Dielectric loss of the capacitors: $Q_d = 100$;
- Frequency at which the above losses are measured: $f_m = 14$ GHz;
- Passband: 7 GHz $\leqslant f \leqslant$ 14 GHz;
- Maximum complexity of the matching networks:
  input matching network: $n = 4$, $k = 1$;
  interstage matching network: $n = 4$, $k = 1$;
  output matching network: $n = 3$, $k = 1$;
- Flat gain level to be approximated: $T_{01}(\omega) = 4.77$ dB and $T_{02}(\omega) = 9.54$ dB;
- Scattering parameters of the FET's are the same as in [1].

The results of the optimization are as follows:

$$\tilde{e}_{11,1}(s) = \frac{-1.014 - 0.8305s - 1.332s^2 - 1.804s^3}{1.014 + 1.9810s + 2.434s^2 + 1.804s^3} \tag{19a}$$

$$\tilde{e}_{11,2}(s) = \frac{-0.9452 + 0.2347s + 2.240s^2 + 0.9348s^3 + 5.9s^4}{0.9452 + 4.1910s + 6.495s^2 + 7.1460s^3 + 5.9s^4} \tag{19b}$$

$$\tilde{e}_{11,3}(s) = \frac{-0.6746 + 0.8165s - 0.6979s^2 + 2.03s^3}{0.6746 + 1.9370s + 2.2440s^2 + 2.03s^3}. \tag{19c}$$

The two-stage amplifier is shown in Fig. 2 and its performance is given by $T(\omega) = 8.905 + / - 1.235$ dB, which is shown in Fig. 3.

The advantages of the real frequency technique are clearly shown in the example. The program can automatically find optimal topologies for the lossy matching networks within the maximum degree of the denominator n when k, which specifies the order of transmission zeros, is fixed (e.g., the actual degree of the denominator of the (19a) is 3 rather than the 4 specified). The synthesis of the matching networks is carried out along with the numerical work by continuous fraction expansion of the input impedances which correspond to the (19a), (19b), and (19c). The final input bandpass matching network with four elements, in which the ideal transformer is canceled, can be obtained by the well-known Norton transformation. Thus the interstage and the output bandpass matching networks can be done with five and four elements.

V. Discussion

This example is similar to that of [1], but the computational steps are quite simplified. Our method can not only synthesize the lossy matching networks directly; it can also employ any complex models of lumped elements with arbitrary nonuniform losses, so that the performance of the amplifier can approach the actual one very well from a theoretical point of view.

Appendix I
Proof of The Theorem

It is assumed that (2) exists if the condition in the theorem is satisfied. By dividing both sides of (2) by $\sqrt{Z_1 Z_2}$, we have a new impedance, $\tilde{Z}$, expressed as

$$\tilde{Z} = Z/\sqrt{Z_1 Z_2} = \sqrt{Z_2/Z_1}\, f(Z_1/Z_2) \tag{A1}$$

Let $\sqrt{Z_1/Z_2}$ be equal to a new variable Ω; then the above equation will have the following form.

$$\tilde{Z} = (1/\Omega) f(\Omega^2). \tag{A2}$$

If Ω is considered an equivalent complex angular frequency, we will have an equivalent lossless inductance $\tilde{Z}_l = \Omega$ and capacitance $\tilde{Z}_c = 1/\Omega$. Then, (A2) may be written as

$$\tilde{Z} = \tilde{Z}_c f(\tilde{Z}_l/\tilde{Z}_c) \tag{A3}$$

which has the same form as (2). In comparison with impedance Z in (2), $\tilde{Z}$ can be regarded as the lossless impedance of the network $\boldsymbol{M}$, which is transformed from the lossy or lossless network $\boldsymbol{N}$ and is composed of lossless inductances and capacitances proportional to $\tilde{Z}_l$ and $\tilde{Z}_c$, respectively.

By a similar procedure, we can also get the following impedance transformation from the lossless network $\boldsymbol{M}$ to the lossy or lossless network $\boldsymbol{N}$.

$$Z = \sqrt{Z_1 Z_2}\,\tilde{Z} = Z_2 f(Z_1/Z_2) \tag{A4}$$

The proof is completed.

Appendix II
Proof of Corollary 2

It is known that the relations between the unit normalized scattering and impedance matrices shown below are valid whether the networks $\boldsymbol{N}$ and $\boldsymbol{M}$ are reciprocal or not [17]:

$$\tilde{\boldsymbol{S}} = (\tilde{\boldsymbol{Z}} + \boldsymbol{I})^{-1}(\tilde{\boldsymbol{Z}} - \boldsymbol{I}) = (\tilde{\boldsymbol{Z}} - \boldsymbol{I})(\tilde{\boldsymbol{Z}} + \boldsymbol{I})^{-1} \tag{A5a}$$

$$\boldsymbol{S} = (\boldsymbol{Z} + \boldsymbol{I})^{-1}(\boldsymbol{Z} - \boldsymbol{I}) = (\boldsymbol{Z} - \boldsymbol{I})(\boldsymbol{Z} + \boldsymbol{I})^{-1} \tag{A5b}$$

$$\tilde{\boldsymbol{Z}} = (\boldsymbol{I} - \tilde{\boldsymbol{S}})^{-1}(\boldsymbol{I} + \tilde{\boldsymbol{S}}) = (\boldsymbol{I} + \tilde{\boldsymbol{S}})(\boldsymbol{I} - \tilde{\boldsymbol{S}})^{-1} \tag{A6a}$$

$$\boldsymbol{Z} = (\boldsymbol{I} - \boldsymbol{S})^{-1}(\boldsymbol{I} + \boldsymbol{S}) = (\boldsymbol{I} + \boldsymbol{S})(\boldsymbol{I} - \boldsymbol{S})^{-1}. \tag{A6b}$$

Employing the (10b) in (A5b) and then substituting (A6a) in it, we obtain

$$\boldsymbol{S} = \left[\sqrt{Z_1 Z_2}(\boldsymbol{I} + \tilde{\boldsymbol{S}}) - (\boldsymbol{I} - \tilde{\boldsymbol{S}})\right] \cdot \left[\sqrt{Z_1 Z_2}(\boldsymbol{I} + \tilde{\boldsymbol{S}}) + (\boldsymbol{I} - \tilde{\boldsymbol{S}})\right]^{-1}. \tag{A7a}$$

The inverse transformation of (A7a) from $\boldsymbol{S}$ to $\tilde{\boldsymbol{S}}$ can be obtained in the same manner:

$$\tilde{\boldsymbol{S}} = \left[(\boldsymbol{I} + \boldsymbol{S}) - \sqrt{Z_1 Z_2}(\boldsymbol{I} - \boldsymbol{S})\right] \cdot \left[(\boldsymbol{I} + \boldsymbol{S}) + \sqrt{Z_1 Z_2}(\boldsymbol{I} - \boldsymbol{S})\right]^{-1}. \tag{A7b}$$

The proof is completed.

Appendix III
Derivation of the Expressions for the TPG

Considering Fig. 1, we can rewrite the definition of T_k as follows:

$$T_k = \frac{\text{power available from FET}_k}{\text{power available from FET}_{k-1}} = \frac{\text{power available from } N_k}{\text{power availble from FET}_{k-1}} \times \frac{\text{power available from FET}_k}{\text{power available from } N_k} = T_{k,1} T_{k,2} \tag{A8}$$

where $T_{k,1}$ is given in detail by

$$T_{k,1} = \frac{\text{power delivered to } N_k \text{ when terminated with } R_{ls}}{\text{power available from FET}_{k-1}}$$

$$\times \frac{\text{power available from } E_{gs}}{\text{power delivered to } N_k \text{ when terminated with } R_{ls}}$$

$$\times \frac{\text{power delivered to } R_{ls} \text{ terminating } N_k}{\text{power available from } E_{gs}}$$

$$\times \frac{\text{power available from } N_k}{\text{power delivered to } R_{ls} \text{ terminating } N_k}$$

$$= \frac{(1-|\Gamma_{g,k}|^2)(1-|e_{11,k}|^2)}{|1-\Gamma_{g,k}e_{11,k}|^2}$$

$$\times \frac{1}{(1-|e_{11,k}|^2)}|e_{21,k}|^2\frac{1}{(1-|\Gamma_k|^2)}$$

$$= \frac{(1-|\Gamma_{g,k}|^2)|e_{21,k}|^2}{|1-\Gamma_{g,k}e_{11,k}|^2(1-|\Gamma_k|^2)}. \quad \text{(A9)}$$

Here R_{ls} and E_{gs} are the supposed 1Ω load and generator. Note that $T_{k,1}$ depends only on N_k and has nothing to do with the following devices and networks if $\Gamma_{g,k}$ is known.

Since $T_{k,2}$ is similar to $T_{k,1}$, its expression can be easily written as

$$T_{k,2} = \frac{(1-|\Gamma_k|^2)|s_{21,k}|^2}{|1-\Gamma_k s_{11,k}|(1-|\Gamma_{g,k+1}|^2)}. \quad \text{(A10)}$$

For the *TPG* of the last stage, it can be redefined as

$$T_{\tilde{N}+1} = \frac{\text{power available from } N_{\tilde{N}+1}}{\text{power available from FET}_{\tilde{N}}}$$

$$\times \frac{\text{power delivered to load } Z_l}{\text{power available from } N_{\tilde{N}+1}} = T_{\tilde{N}+1,1}T_{\tilde{N}+1,2} \quad \text{(A11)}$$

where $T_{\tilde{N}+1,1}$ is equal to $T_{k,1}$ when $k = \tilde{N}+1$, and $T_{\tilde{N}+1,2}$ is given by

$$T_{\tilde{N}+1,2} = 1 - |\Gamma_l|^2. \quad \text{(A12)}$$

Therefore, by multiplying $T_{\tilde{N}+1,1}$ by $T_{\tilde{N}+1,2}$, we obtain the expression for $T_{\tilde{N}+1}$. The derivation is completed.

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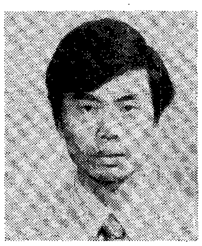

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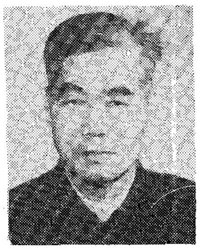

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